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Parameter Identification of an Antiaircraft Gunner Model

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A systematic procedure is presented to determine all the parameters of an antiaircraft gunner model designed by the authors. The combined least-squares curve-fitting method and the modified Gauss-Newton gradient algorithm are used to develop the parameter identification program. With this program, consistent parameter values of the gunner model are obtained from various initial guesses. There is no need for trial-and-error manual tuning of parameter values. The identification program provides a convenient and accurate procedure to determine gunner model parameters. Computer simulation results show that the prediction of tracking errors by the gunner model is a good description of actual gunner's tracking performance. The gunner model and the parameter identification procedure have been successfully utilized to study the effectiveness of several air defense weapon systems.

Nomenclature

A,F,D = coefficient matrices of the plant in Eq. (1)
a = vector of model parameters
a_0 = initial guess of model parameters
c = weighting factor in the criterion function
e_T = tracking error $(\theta_T - \theta_g)$
$E[\cdot]$ = expectation value of $\cdot$
J = scalar criterion function
k = observer gain
P = covariance matrix of X
P_n = covariance matrix of $X(t_n)$
P_{11} = first diagonal element of P
$\bar{s}$ = ensemble standard deviation of tracking error
$\bar{s}^I$ = averaged reference standard deviation of empirical tracking error
A^T = transpose of matrix A
t = time
t_n = integer n times Δ_t
t_f = tracking duration
Δ_t = sampling period
u = control output of the gunner model
u_c = output of the feedback controller
v = remnant element
v_n = remnant at t_n
X = state vector of the antiaircraft artillery tracking system
$\dot{X}$ = time derivative of X
x_1,x_2,x_3 = state components of X in Eq. (2)
$\bar{X}$ = expectation value of X
$\dot{\bar{X}}$ = time derivative of $\bar{X}$
$\bar{x}_1$ = first state component of $\bar{X}$, i.e., ensemble mean of tracking error
$\bar{x}_1^I$ = reference empirical mean of tracking error
X_n = state vector X at t_n
$\bar{X}_n$ = expectation value of X_n
a_1,a_2,a_3 = coefficients of remnant covariance function
$\gamma_1\gamma_2$ = feedback controller gains
θ_T = target elevation angle
$\dot{\theta}_T$ = target elevation angle rate
$\ddot{\theta}_T$ = target elevation angular acceleration
$\hat{\dot{\theta}}_T$ = estimate of $\dot{\theta}_T$
$\hat{\ddot{\theta}}_T$ = estimate of $\ddot{\theta}_T$
$\ddot{\theta}_{T,n}$ = target elevation angular acceleration at t_n
θ_g = gunsight elevation angle
$\delta(t)$ = Dirac delta function
τ = time
ϕ,Γ_1,Γ_2 = coefficient matrices of the plant in Eq. (11)

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Introduction

A SYSTEMATIC study of air defense weapon effectiveness and aircraft attrition in ground-to-air combat requires the development of a computer program for simulating engagement dynamic systems. One of the important parts of such a program is a mathematical model describing human tracking performance in operating antiaircraft artillery (AAA) systems. The structure of a mathematical human operator model (also called a gunner model[1]) was designed by the authors for this purpose. It is composed of three main elements: a reduced-order observer,[2] a linear state variable feedback controller, and a random remnant. The reduced-order observer is used to describe human estimation of those unmeasurable state variables of AAA systems. The feedback controller represents human tracking function to align the gunsight line angle with the target position angle. All the effects of various randomness sources in the AAA man-machine closed-loop system and of the modelling errors are equivalently lumped into one remnant element in this design. These effects include the observation error, the neuromotor error, target uncertainty modelling error, etc. There are several undetermined parameters in the three main elements of the gunner model. In this paper, a parameter identification program is developed to determine these parameters. The combined least-squares curve-fitting method and the modified Gauss-Newton gradient iterative algorithm are used in the design of the program. The criterion function of the combined least-squares curve-fitting method is selected such that model predictions of both ensemble mean and standard deviation of tracking errors are curve-fitting to averaged empirical data of the manned AAA simulation experiments.[3] Then, by the modified Gauss-Newton algorithm, the parameter values of the gunner model are iteratively adjusted until the model predictions of azimuth and elevation tracking errors match the empirical data. This identification program provides a convenient procedure to determine the parameters of the gunner model for AAA weapon systems. There is no need to find model parameters by time-consuming manual trial-and-error tuning. The parameter identification procedure is especially useful in the weapon effectiveness study of many various AAA systems. A set of parameter values of the gunner model can be easily obtained for each AAA weapon system. This is one of the highlights of this model. A computer simulation program for the closed-loop AAA tracking task is also developed. Simulation results are compared with the corresponding experimental data for various flyby and maneuvering trajectories. Model predictions of the ensemble mean and the ensemble standard deviation of tracking errors are shown to be good representations of actual gunner tracking data.

Systems Description and Gunner Model

The block diagram of an AAA tracking system is shown in Fig. 1 in which the gunner model represents the gunner's compensatory tracking response. The detailed mathematical models of the gunsight system and the target motion and the equations of the gunner model (reduced-order observer, state variable feedback controller, and remnant element) can be found in Ref. 1. The purpose of this paper is to present a systematic procedure to determine the model parameters. Then, the model can be used in a computer simulation program of a closed-loop AAA tracking system to generate the gunner's response as accurately as possible. In order to show the parameter identification procedure, the elevation tracking dynamic equations of a specific closed-loop AAA weapon system will be written in the following manner (which can be obtained from the general dynamic equations in Ref. 1):

$$\dot{X} = AX + F\ddot{\theta}_T + Dv \tag{1}$$

where X is a three-dimensional state vector of the closed-loop AAA tracking system

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} e_T \\ \dot{\theta}_T - ke_T \\ \dot{\theta}_T - \hat{\dot{\theta}}_T \end{bmatrix} \tag{2}$$

e_T is the tracking error and equals the difference between target elevation angle θ_T and gunsight elevation θ_g. k is the observer gain which is one of the model parameters; $\dot{\theta}_T$ and $\hat{\dot{\theta}}_T$ are the target elevation angle rate and its estimation from the observer. The gunsight system of Fig. 1 is expressed by $\dot{\theta}_g = 1.34u$. Then, the matrices A, F, and D of Eq. (1) are

$$A = \begin{bmatrix} k+1.34(\gamma_1 + k\gamma_2) & 1+1.34\gamma_2 & -1.34\gamma_2 \\ -k^2 - 1.34k(\gamma_1 + k\gamma_2) & -k-1.34k\gamma_2 & 1.34k\gamma_2 \\ 0 & 0 & -k \end{bmatrix}$$

$$F = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \qquad D = \begin{bmatrix} -1.34 \\ 1.34\,k \\ 1.34\,k \end{bmatrix} \tag{3}$$

where γ_1 and γ_2 are two feedback control gains which are also two model parameters. $\ddot{\theta}_T$ and v of Eq. (1) denote the target elevation angle acceleration and the remnant element. The latter is modelled as a Gaussian white noise with the following properties:

$$E[v(t)] = 0 \quad \text{for all } t \tag{4a}$$

and

$$E[v(\tau)v(\tau)] = [a_1 + a_2\hat{\dot{\theta}}_T^2(t) + a_3\hat{\ddot{\theta}}_T^2(t)]\,\delta(t-\tau) \quad \text{for all } t-z \tag{4b}$$

where a_1, a_2, and a_3 are three nonnegative parameters of the model to be determined. $\delta(t)$ is the Dirac delta function, $\hat{\dot{\theta}}_T$ and $\hat{\ddot{\theta}}_T$ are the estimated target angle rate and acceleration, respectively.

Before the gunner model, based on the observer theory, can be used to describe gunner's tracking response, the parameters k, γ_1, γ_2, a_1, a_2, and a_3 of Eqs. (2-4) must be determined. The authors have developed a curve-fitting parameter identification program in Ref. 4 using the least squares method[5] and the Gauss-Newton gradient method.[6] Model predictions of the ensemble mean and the ensemble standard deviation of tracking errors will be used in the curve-fitting program. Therefore, they are derived in terms of model parameters in the following.

Taking expectation values of both sides of Eq. (1), we have

$$\dot{\bar{X}} = A\bar{X} + F\ddot{\theta}_T \tag{5}$$

where $\bar{X} = E[X]$. The first component $\bar{x}_1$ of $\bar{X}$ is the model prediction of ensemble mean of tracking error. By solving vector differential Eq. (5), $\bar{x}_1$ can be expressed in terms of model parameters.[4] [$\bar{x}_1(0)$ is assumed to be zero.]

$$\bar{x}_1(t) = \int_0^t \frac{k+1.34\gamma_1+1.34K\gamma_2}{1.34\gamma_1(k+1.34\gamma_1)} e^{1.34\gamma_1(t-\tau)} + \frac{1.34\gamma_2}{k+1.34\gamma_1} e^{-k(t-\tau)} - \frac{1+1.34\gamma_2}{1.34\gamma_1} \cdot \ddot{\theta}_T(\tau)\,d\tau \tag{6}$$

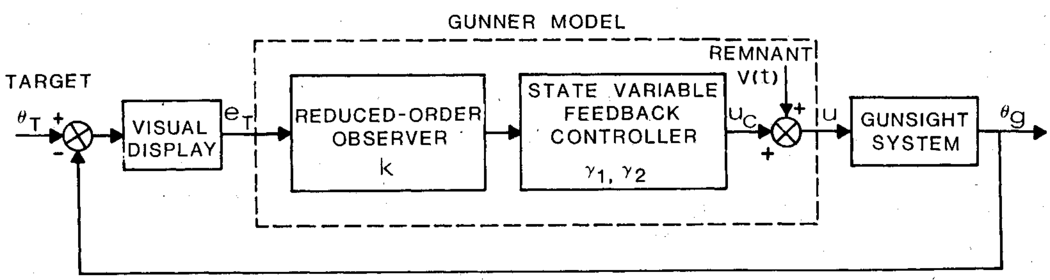

Fig. 1 Block diagram of an AAA system with gunner model.

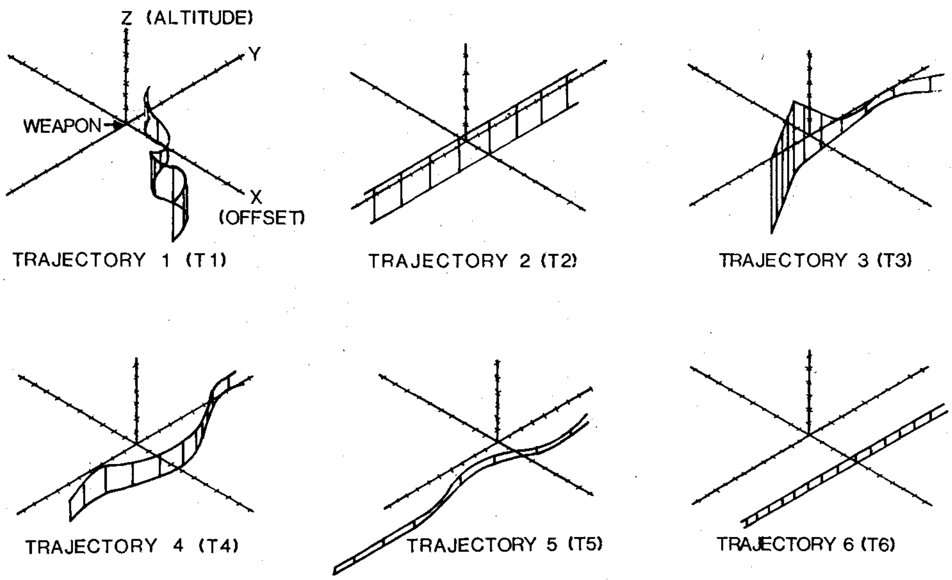

Fig. 2 Block diagram of the parameter identification procedure.

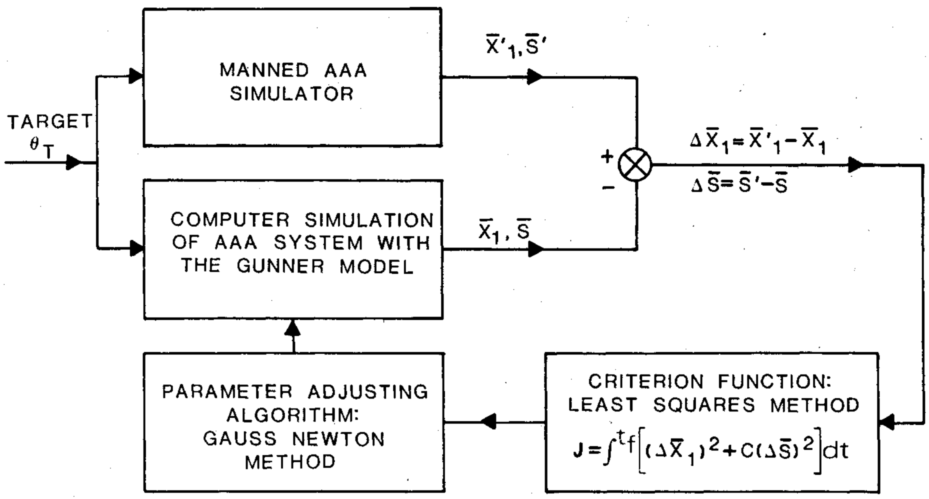

Fig. 3 Flyby and maneuvering target trajectories.

Furthermore, let $P(t) = [E\,(X(t)-\bar{X}(t))\,(X(t)-\bar{X}(t))^T]$, and it satisfies the following covariance equation[7]:

$$\dot{P}=AP+PA^T+D(a_1+a_2\hat{\dot{\theta}}^2_T(t)+a_3\hat{\ddot{\theta}}^2_T(t))D^T \tag{7}$$

By solving this matrix differential equation and with the assumption $p_{11}(0)=0$, the first diagonal element $p_{11}(t)$ of $P(t)$ can be obtained:

$$P_{11}(t)=\int_0^t 1.34^2\left\{\frac{-1}{k+1.34\gamma_1}[1.34k\gamma_2 e^{-k(t-\tau)} - (k+1.34\gamma_1+1.34\gamma_2)\cdot e^{1.34\gamma_1(t-\tau)}]\right\}^2 \cdot (a_1+a_2\hat{\dot{\theta}}^2_T(\tau)+a_3\hat{\ddot{\theta}}^2_T(\tau))\,\mathrm{d}\tau \tag{8}$$

Let

$$\bar{s}(t)=p_{11}(t)^{1/2} \tag{9}$$

then $\bar{s}(t)$ denotes the model prediction of ensemble standard deviation of tracking errors. Equations (6) and (9) are ensemble mean and ensemble standard deviation of tracking errors which are explicit functions of model parameters and will be used in the curve-fitting program in the next section.

Parameter Identification Procedure

Figure 2 shows a block diagram of the combined least-squares curve-fitting identification program. The reference curves in the curve-fitting program are obtained from empirical data of manned AAA simulation experiments. Six simulated flyby and maneuvering aircraft trajectories of 35-s duration of Fig. 3 are used as target trajectories for the preceding experiments. The antiaircraft weapon is located at the origin of the local horizontal X-Y plane, and the Z axis represents the altitude. The increment of each of the three axes is 1000 ft. A detailed description of the characteristics of these trajectories can be found in Ref. 3. Let $\bar{x}_1^I(t)$ and $\bar{s}^I(t)$ be the reference empirical mean and standard deviation of tracking errors which are obtained by averaging the results of twenty-five experimental replication simulation runs with a typical maneuvering trajectory T3 of Fig. 3 as the target input and the same subject. Now the criterion function J of the combined least-squares curve-fitting program is selected as

$$J(a)=\int_0^{t_f}[(\bar{x}_1^I(t)-\bar{x}_1(t,a))^2+c(\bar{s}^I(t)-\bar{s}(t,a))^2]\,\mathrm{d}t \tag{10}$$

where t_f is the tracking duration (equal to 35 s in this case). $\bar{x}_1^I$ and $\bar{s}^I$ are empirical reference data, and $\boldsymbol{a}$ is the parameter vector of the gunner model

$$\boldsymbol{a}^T=[k\gamma_1\gamma_2 a_1 a_2 a_3]$$

Table 1 Gunner model parameters

	Observer gain	Controller gains		Coefficients of remnant covariance function		
	k	γ_1	γ_2	α_1	α_2	α_3
Elevation tracking	1.88	−1.99	−0.745	0.0000094	0.025	0.068
Azimuth tracking	5.12	−3.51	−0.762	0.0000363	0.00614	0.0117

$\bar{x}_1$ and $\bar{s}$ are functions of time and parameter vector $\boldsymbol{a}$, as shown in Eqs. (6) and (9); c is a positive weighting factor selected to be one in this case. The criterion function J of Eq. (10) can be considered as a combination of two integrations. One of them is an integration of the square of the error between the empirical mean tracking error $\bar{x}_1^l$ and the model prediction of the ensemble mean tracking error x_1. The other is a similar integration related to the standard deviation of tracking errors. Therefore, minimizing J with respect to the parameter vector $\boldsymbol{a}$ is called a combined least-squares curve-fitting method. We would like to point out that in this method, model parameters will be determined such that empirical mean and standard deviation of tracking errors are fitted simultaneously by the corresponding model prediction functions x_1 and s. Now the parameter identification task becomes a minimization problem, i.e., to find values of the parameter vector $\boldsymbol{a}$ which minimize the criterion function $J(\boldsymbol{a})$ of Eq. (10). The modified Gauss-Newton iterative gradient method, derived in the Appendix, iteratively adjusts parameter values to minimize the criterion function J. This iterative process will continue until the increments are smaller than a preassigned lower bound. A computer curve-fitting program is developed using the previously mentioned methods. It provides a convenient and accurate procedure to determine gunner model parameters. The parameter values[4] of the gunner model determined by this identification procedure for the trajectory T3 are listed in Table 1.

From the manned AAA simulation experiments, it is found that gunner's tracking response and tracking error in the azimuth axis are different from those in the elevation axis. Table 1 also shows that the model parameters for the azimuth tracking and for the elevation tracking are different.

Furthermore, we would like to point out that for each of these two cases, the combined curve-fitting parameter identification program always converges to the same set of parameter values from various initial guesses. In addition, the parameter values of the gunner model depend on the dynamics of AAA gunsight systems. For different AAA gunsight dynamic systems, the combined curve-fitting program will determine different parameter values for the gunner model.

Computer Simulation Results

Once the model parameters are determined, the gunner model is ready to be used in a computer simulation of the AAA closed-loop tracking system to describe the gunner's tracking performance for the maneuvering trajectory T3. The parameter values are then substituted into the elements of matrices $\boldsymbol{A}$, $\boldsymbol{F}$, and $\boldsymbol{D}$ of Eq. (1), which is a mathematical dynamic model of the overall closed-loop AAA tracking system. In order to find a solution of Eq. (1) without using convolution integrations, it can be discretized according to Ref. 7 to be

$$X_{n+1} = \phi X_n + \Gamma_1 \ddot{\theta}_{T,n} + \Gamma_2 v_n \tag{11}$$

where

$$X_{n+1} = X(t_{n+1})$$

with

$$t_{n+1} = (n+1)\cdot\Delta t \quad \text{and} \quad \Delta t = 0.066\ \text{s}$$

$$\phi = \exp[A\Delta t] \qquad \Gamma_1 = \int_0^{\Delta t} \exp[A\sigma]\,d\sigma\cdot \boldsymbol{F} \tag{12}$$

$$\Gamma_2 = \int_0^{\Delta t} \exp[A\sigma]\,d\sigma\cdot \boldsymbol{D} \qquad \ddot{\theta}_{T,n} = \ddot{\theta}_T(t_n) \tag{13}$$

and v_n is a random sequence with the following properties:

$$\mathrm{E}[v_n] = 0$$

$$\mathrm{E}[(v_n)(v_n)T] = 1/\Delta t(a_1 + a_2\hat{\dot{\theta}}_T^2(t_n) + a_3\hat{\ddot{\theta}}_T^2(t_n))$$

Taking expectation values of both sides of Eq. (11),

$$\bar{X}_{n+1} = \phi\bar{X}_n + \Gamma_1\ddot{\theta}_{T,n} \tag{14}$$

Let P_{n+1} denote the covariance of X_{n+1}, then it can be shown[7] that P_{n+1} satisfies the following matrix difference equation:

$$P_{n+1} = \phi P_n \phi^T + \Gamma_2(1/\Delta t)(a_1 + a_2\hat{\dot{\theta}}_T^2(t_n) + a_3\hat{\ddot{\theta}}_T^2(t_n))\Gamma_2^T \tag{15}$$

Then, the first element $\bar{X}_{n+1}$ of Eq. (14) and the square root of the first diagonal element of the matrix P_{n+1} of Eq. (15) are the model predictions of the ensemble mean and the ensemble standard deviation of tracking errors, respectively. A computer program simulating the AAA closed-loop tracking system is developed using the preceding recursive Eqs. (14) and (15). The input to the program is the target maneuvering trajectory T3 of Fig. 3. The output of the computer simulation program are ensemble mean and ensemble standard deviation of tracking errors which can be used to predict the gunner's tracking characteristics. Computer simulation results of the AAA tracking system show that model predictions match well with the empirical data for both mean and standard deviation of tracking errors. Although only the elevation tracking system equations are described in this paper, the authors have also studied the corresponding azimuth case.[1] Typical simulation results for both the elevation tracking and the azimuth tracking are shown in Figs. 4-7. The solid curves in these figures represent the averaged empirical data of tracking errors over twenty-five replications from manned AAA simulation experiments. The corresponding dashed curves denote the model prediction of tracking errors obtained from digital computer simulation runs. The simulated target trajectory T3 crosses over the AAA weapon system around the 25th second. It is a difficult tracking period since the magnitudes of target elevation and azimuth angle rates are large. Therefore, in each of these four figures, the tracking errors around the 25th s are much larger than the rest of the tracking error curve. The model predictions of both the mean curves and the standard deviation curves match well with the averaged empirical data over twenty-five replications. So far, only the maneuvering trajectory T3 was used for both the parameter identification program and the AAA computer simulation program. Next,

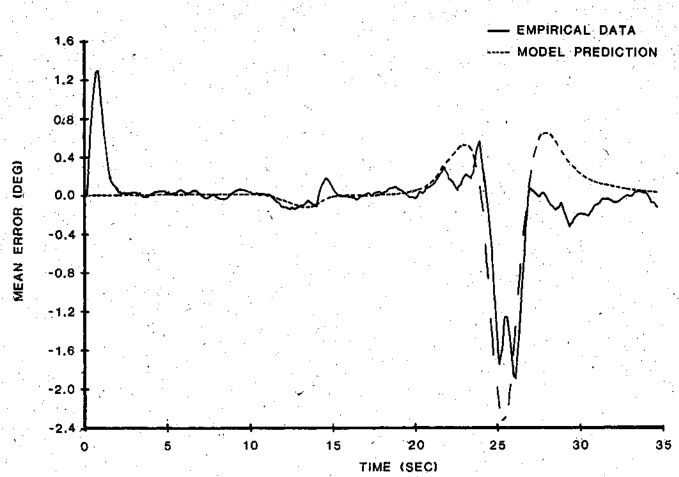

Fig. 4 Mean tracking error, elevation T3.

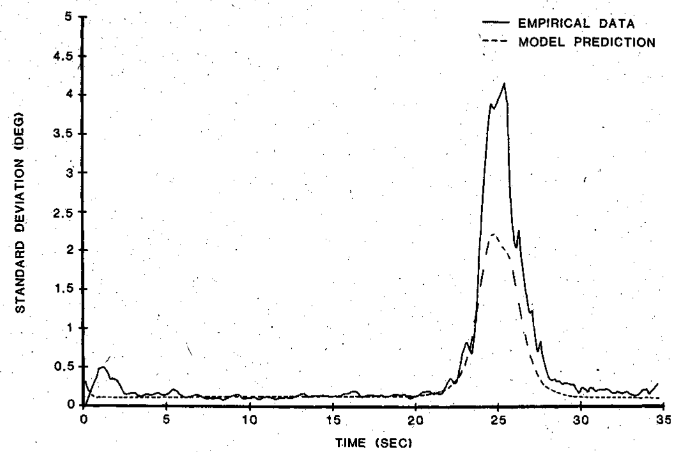

Fig. 7 Standard deviation of tracking error, azimuth T3.

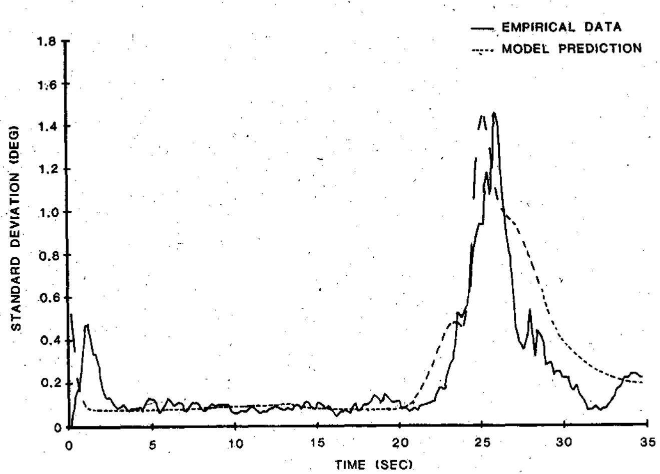

Fig. 5 Standard deviation of tracking error, elevation T3.

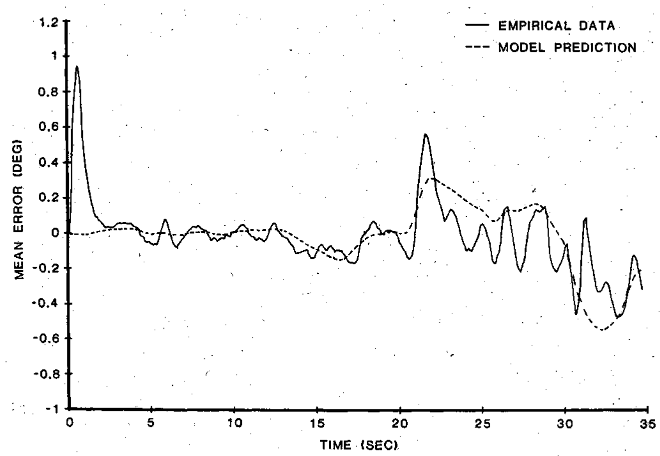

Fig. 8 Mean tracking error, elevation T1.

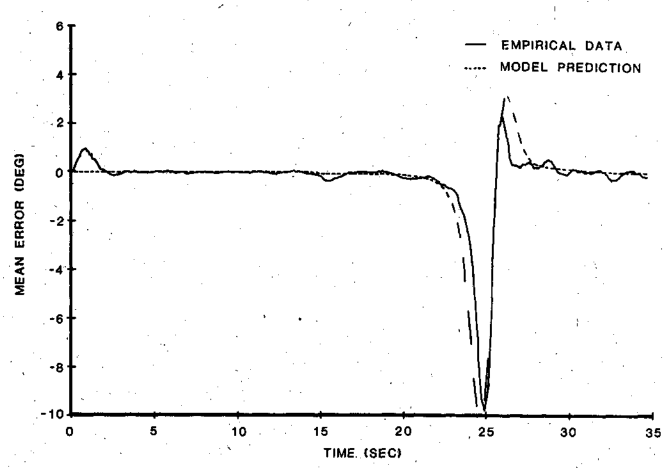

Fig. 6 Mean tracking error, azimuth T3.

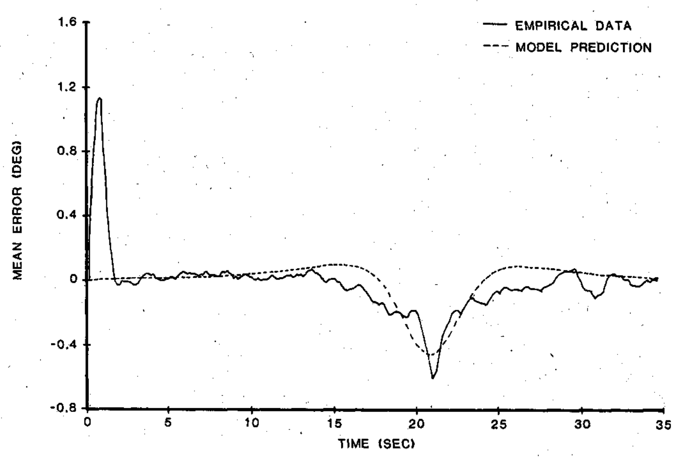

Fig. 9 Mean tracking error, elevation T2.

we would like to point out another highlight of the simulation work. The set of model parameter values, which was obtained with respect to trajectory T3, is used in the computer simulation program to obtain model predictions for various trajectories of Fig. 3 which are typical flight trajectories of a certain type of fighter. The results are shown in Figs. 8-12 where the generated elevation mean tracking errors for all other flyby and maneuvering trajectories also match well with the corresponding empirical data. In addition to the elevation mean tracking errors, model predictions of elevation standard deviation, azimuth mean, and azimuth standard deviation of tracking errors for all these trajectories of Fig. 3 are in agreement with empirical data. Therefore, it is found from the computer simulation experience that the parameters of the gunner model for a given AAA weapon system are not sensitive with respect to target trajectories. It has been shown that a same set of parameter values for the gunner model can be used to generate model outputs representing human tracking responses for various practical aircraft trajectories. This gunner model is then called a predictive model in the sense that once the model parameters are determined, the gunner can be used to predict gunner tracking responses for other trajectories.

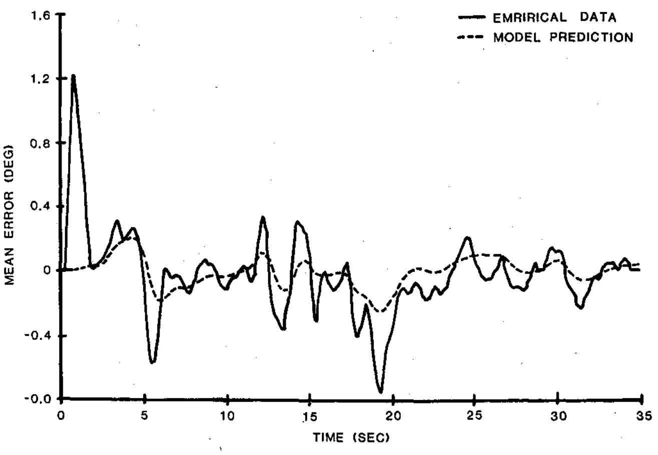

Fig. 10 Mean tracking error, elevation T4.

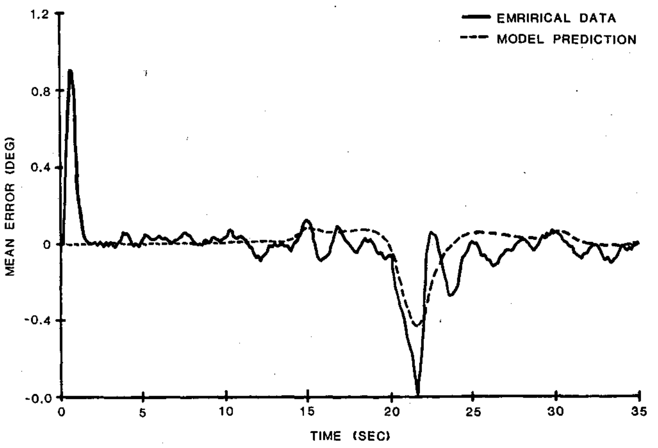

Fig. 11 Mean tracking error, elevation T5.

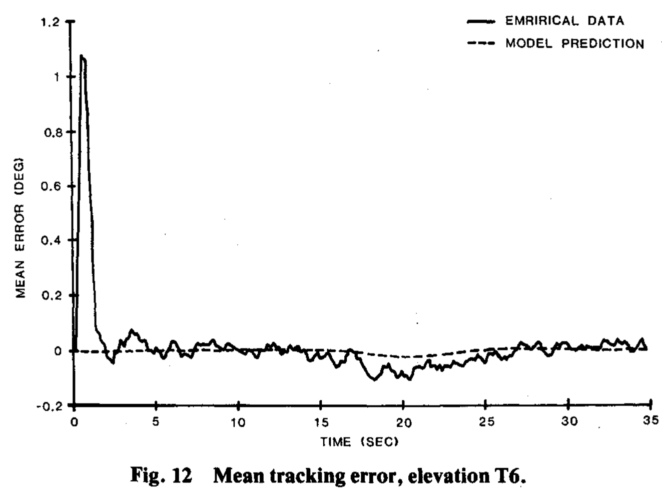

Fig. 12 Mean tracking error, elevation T6.

Conclusion

This paper has presented a convenient and accurate procedure to determine parameter values of the gunner model. It has been shown that for a given antiaircraft artillery (AAA) weapon system, a set of model parameter values can be systematically obtained by the combined least-square curve-fitting identification program. Computer simulation results have shown that the same set of parameter values can be used in the gunner model to generate gunner tracking responses for various flyby and maneuvering trajectories. Digital computer simulation results for both elevation and azimuth tracking cases match very well with the corresponding empirical data from manned AAA simulation experiments. So, the antiaircraft gunner model is a reliable mathematical representation of the actual gunner tracking response in operating an AAA weapon system. The gunner model and its predictive property provide a useful tool for the simulation study of air defense weapon effectiveness, aircraft attrition estimate, and ground-to-air engagement systems. This model has been successfully utilized to study the effectiveness of several foreign air defense weapon systems. A set of parameter values of the gunner model was obtained for each of these weapon systems by the presented parameter identification program. All the simulation results showed that only one set of parameter values for each weapon system is necessary to generate model predictions of tracking errors for various practical trajectories. The limitations of the predictive property of the gunner model with respect to trajectories for other types of fighters are now under investigation.

Appendix: Derivation of the Iterative Algorithm of the Modified Gauss-Newton Method

The criterion function $J(a)$ of Eq. (10) is rewritten here:

$$J(a)=\int_0^{t_f}[(\bar{x}_I'(t)-\bar{x}_I(t,a))^2+c(\bar{s}'(t)-\bar{s}(t,a))^2]\,\mathrm{d}t$$

Taking first-order Taylor series expansions of $\bar{x}_I$ and $\bar{s}$ with respect to a certain initial guess a_0, we have

$$J(a)\cong\int_0^{t_f}\left\{\left[\bar{x}_I'(t)-\bar{x}_I(t,a_0)-\frac{\partial\bar{x}_I(t,a_0)}{\partial a}\cdot(a-a_0)\right]^2\right.$$
$$\left.+c\left[\bar{s}'(t)-\bar{s}(t,a_0)-\frac{\partial\bar{s}(t,a_0)}{\partial a}\cdot(a-a_0)\right]^2\right\}\mathrm{d}t$$

Next, the partial derivative of J with respect to a can be found as

$$\frac{\partial J(a)}{\partial a}\cong\int_0^{t_f}\left\{-2(\bar{x}_I(t)-\bar{x}_I(t,a_0))\frac{\partial\bar{x}_I(t,a_0)}{\partial a}\right.$$
$$+2\frac{\partial\bar{x}_I(t,a_0)}{\partial a}(a-a_0)\cdot\frac{\partial\bar{x}_I^T(t,a_0)}{\partial a}$$
$$+c\left[-2(\bar{s}'(t)-\bar{s}(t,a_0))\frac{\partial\bar{s}(t,a_0)}{\partial a}\right.$$
$$\left.\left.+2\frac{\partial\bar{s}(t,a_0)}{\partial a}(a-a_0)\frac{\partial\bar{s}^T(t,a_0)}{\partial a}\right]\right\}\mathrm{d}t \qquad \text{(A1)}$$

Assume that a^* is a minimal, then

$$\frac{\partial J(a^*)}{\partial a}=0 \qquad \text{(A2)}$$

After discretization, Eqs. (A1) and (A2) can be rearranged to be

$$a^*=a_0-\left[\sum_{k=1}^{K}2\left(\frac{\partial\bar{x}_I}{\partial a}\right)^T\left(\frac{\partial\bar{x}_I}{\partial a}\right)+c_2\left(\frac{\partial\bar{s}}{\partial a}\right)^T\left(\frac{\partial\bar{s}}{\partial a}\right)\right]^{-1}$$
$$\times\sum_{k=1}^{K}\left\{-2(\bar{x}_I'(t)-\bar{x}_I)\left(\frac{\partial\bar{x}_I}{\partial a}\right)^T\right.$$
$$\left.+c\left[-2(\bar{s}'-\bar{s})\left(\frac{\partial\bar{s}}{\partial a}\right)^T\right]\right\} \qquad \text{(A3)}$$

Equation (A3) can be extended to a more general form

$$a_{i+1}=a_i-\left[\sum_{k=1}^{K}2\left(\frac{\partial\bar{x}_1(t_k,a_i)}{\partial a}\right)^T\left(\frac{\partial x_1(t_k,a_i)}{\partial a}\right)\right.$$
$$\left.+c_2\cdot\left(\frac{\partial\bar{s}(t_k,a_i)}{\partial a}\right)^T\left(\frac{\partial\bar{s}(t_k,a_i)}{\partial a}\right)\right]^{-1}$$
$$\times\sum_{k=1}^{K}\left\{-2(\bar{x}_1'(t_k)-\bar{x}_1(t_k,a_i))\left(\frac{\partial\bar{x}_1(t_k,a_i)}{\partial a}\right)^T\right.$$
$$\left.+c\left[-2(\bar{s}'(t_k)-\bar{s}(t_k,a_i))\left(\frac{\partial\bar{s}(t_k,a_i)}{\partial a}\right)^T\right]\right\}$$
$$(i=0,1,2,3,\ldots)$$

This is the modified Gauss-Newton iterative gradient algorithm.

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